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BPS pp-wave brane cosmological solutions in string theory

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Abstract: We construct time dependent BPS pp-wave brane solutions in the context of M-theory and type II supergravity. It is found that the N-brane solutions we consider satisfy the crossing rule as S-brane solutions but 1/8 supersymmetry remains. By applying them to the cosmological setting, inflationary solutions are obtained. During this inflation, the size of the extradimensions becomes smaller than our four-dimensional spacetime dynamically. We also discuss the mechanism for terminating this inflation and recovering the hot big-bang universe.

Keywords: p-branes, M-Theory, Supersymmetry Phenomenology, Cosmology of Theories beyond the SM.

Contents

1. Introduction **1**

2. Model and basic equations **2**

3. PP-wave solutions **3**

3.1 Bosonic part of the solutions and crossing rule 4

3.2 BPS pp-wave brane solutions 7

4. Cosmological solution **9**

5. Conclusion **11**

1. Introduction

With the strong support of observational results, the standard big-bang cosmology supplemented by the inflationary scenario has come to rise in status [1]. What is necessary next is to explain the corresponding cosmological solutions in the context of a fundamental theory such as string theory. Regardless of the conceptual worries in the study of dS or eternally accelerating universe quantum gravity [2, 3], one possibility to explain de Sitter vaccua [4] as well as inflationary solutions [5] has recently been proposed in the context of string theory. It is known that the no-go theorem of [6] guarantees that such solutions cannot be obtained in string or M theory by using only the lowest-order terms in the 10D or 11D supergravity action. The crucial elements to invalidating the no-go theorem are the D-branes and fluxes supported by form fields in warped backgrounds and allow one to find highly warped compactification such as [7].

D-branes [8] have played a very important role not only in cosmology but also in our understanding of non-perturbative aspects of string or M theory and the AdS/CFT correspondence [9]. As is well known, D-branes can be described as hypersurfaces where open strings can end, which is achieved by imposing Dirichlet boundary conditions along transverse spacelike directions in the string world-sheet action, in perturbative string theory at weak string coupling. The more general situations with D-branes can be understood as intersecting D-branes, with which explicit string theory compactifications have appeared in [10]. What is interesting about the intersecting branes phenomenologically is pointed out in [11]; when D-branes intersect at non-vanishing angles, open strings stretched out between them gives rise to chiral fermions living at the intersection.

Naturally, in string perturbation theory, one can also consider open strings obeying Dirichlet boundary conditions along time-like or null directions. These are space-like or null

JHEP05(2005)016

analogs of D-branes and are called S-branes [12] or N-branes [13], respectively. Until now, even though time-dependent brane solutions can be considered in both cases S-brane solutions had acquired much more attention because of their possible connection with rolling tachyon and dS/CFT correspondence [3, 12, 14 − 20] as well as inflationary solutions [21 − 23] (see also refs. [24, 25] for related solutions). As in D-brane cases, intersecting S-brane solutions can be also obtained [26 − 28]. On the other hand, N-brane solutions are also interesting from the viewpoint of closed/open string correspondence and stringy explanation of the black holes as discussed in ref. [13], where such solutions were discussed in the string worldsheet picture. Recently, some class of explicit intersecting N-brane solutions in supergravity have been obtained [29] in which the intersection rules for the way the solutions can intersect with each other is given based on the method of [28, 30]. The main purpose of this paper is to construct another class of N-brane cosmological solutions that are reminiscent of a stringy set-up.

In order to start with the string theory background, we adopt pp-wave spacetime, which yields exact classical backgrounds for string theory for some time, with all α' corrections vanishing [31, 32]. It is also worth noting that these backgrounds are exactly solvable in the light cone gauge [33 − 35]. The outline of our paper is as follows. In section 2 after presenting the low-energy effective action corresponding to superstring theory in any spacetime dimension, we obtain the basic equations. In section 3 we assume the pp-wave background and the gauge field strength, and find the 1/8 supersymmetric BPS solutions. We also find that the crossing rule and the harmonic rule are satisfied under the metric ansatz. We also indicate the solutions in M or string theory, and show the relations between S-brane solutions [28] there. In section 4, applying to cosmological setting, we find specific solutions corresponding to the inflationary solutions in four dimensions. These solutions provide dynamical compactification naturally. Section 5 is devoted to summary and conclusion as well as the discussion how to end the inflation via supersymmetry breaking.

2. Model and basic equations

Consider the following general action for gravity coupled to a dilaton ϕ and m different n_A- form field strengths:

$$S = \frac{1}{16\pi G_D}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\varphi)^2 - \sum_{A=1}^{m}\frac{1}{2\cdot n_A!}e^{a_A\varphi}F_{n_A}^2\right], \tag{2.1}$$

where R is the Ricci scalar with respect to the metric $g_{\mu\nu}$, φ is the dilaton, F_{n_A} is field strengths of arbitrary form degree $n_A \leq D/2$, and a_A is the coupling between the dilaton and the form field. This action describes the bosonic part of $D = 11$ or $D = 10$ supergravities. In addition to this, in general, there may be Chern-Simons terms in the action. However, since they are irrelevant in our following solutions, we omit them by assuming they are frozen.

The equations of motion corresponding to the Einstein equations, the equation of motion for the dilaton, and the Maxwell equations can be written in the following forms, respectively:

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi + \sum_A \Theta_{A\mu\nu}\,, \\ \nabla^2\varphi &= \sum_A \frac{a_A}{2\cdot n_A!}e^{a_A\varphi}F_{n_A}^2\,, \\ \partial_{\mu_1}(\sqrt{-g}e^{a_A\varphi}F^{\mu_1\cdots\mu_{n_A}}) &= 0\,, \end{aligned} \tag{2.2}$$

where $\Theta_{A\mu\nu}$ is the stress-energy tensor corresponding to the n_A-form field, given by

$$\Theta_{A\mu\nu} = \frac{1}{2\cdot n_A!}e^{a_A\varphi}\left[n_A F_\mu{}^{\rho\cdots\sigma}F_{\nu\rho\cdots\sigma} - \frac{n_A-1}{D-2}F_{n_A}^2 g_{\mu\nu}\right], \tag{2.3}$$

and ∇^2 is a D-dimensional Laplacian with respect to $g_{\mu\nu}$.

The Bianchi identities for the n_A-form serve as the constraint equation, which is

$$\partial_{[\mu}F_{\mu_1\cdots\mu_{n_A}]} = 0\,, \tag{2.4}$$

as they are the field strength of (n_A-1)-form potentials.

3. PP-wave solutions

In this section we find the pp-wave background solution for the basic equaions (2.2). We take the following metric for our system:

$$ds^2 = 2e^{2\xi}du(dv+fdu) + \sum_{i=1}^{d-1}e^{2\eta}dx_i^2 + \sum_{\alpha=2}^{p}e^{2\zeta_\alpha}dy_\alpha^2\,, \tag{3.1}$$

where we have used the light-cone coordinate $u = -(t-y_1)/\sqrt{2}$ and $v = (t+y_1)/\sqrt{2}$. Furthermore, as for the number of spacetime dimensions, we separate them into $D = d+p$, the coordinates y_α, $\alpha = 2,...,p$ parametrize the $(p-1)$-dimensional world-volume directions, and the remaining coordinates of the $(d+1)$-dimensional spacetime are coordinates on $(d-1)$-dimensional flat spaces, u and v. Since we are interested in time-dependent solutions, all the functions appearing in the metrics as well as the dilaton φ are assumed to depend only on the light-cone coordinates u and v. These solutions about the light cone coordinates are named “null-branes”(N-branes) and the Nq_A-brane whose world-volume is (q_A+1)-dimensional, tangential to u, and q_A- spacelike directions. (We assume that these spatial coordinates correspond to some of $\{y_1,\cdots,y_p\}$ in our solutions.)

As for the form fields, the most general ones consistent with the field equations and Bianchi identities should be taken. For this purpose, we assume an electrically charged Nq_A- brane whose value is given by

$$F_{n_A} = d\phi_{q_A} = \partial_v E_{q_A} du\wedge dv\wedge dy_2\wedge\cdots\wedge dy_{q_A+1}\,, \tag{3.2}$$

where $n_A = q_A + 2$ and E_{q_A} is a function of u and v. We can also discuss the magnetic case with a N$\tilde{q_A}$-brane, which is obtained from a dual transformation of the electrically charged Nq_A-brane as

$$\begin{aligned}\tilde{F}_{n_A} &= e^{-a_A\varphi}\partial_v E_{\tilde{q}_A} e^{-2\xi-\sum_\alpha \zeta_\alpha}\epsilon^{uv\alpha_2\cdots\alpha_{\tilde{q}_A+1}}{}_{1\cdots(d-1)\alpha_{\tilde{q}_A+2}\cdots\alpha_p} dx^1\wedge\cdots\wedge dx^{d-1}\wedge \\ &\quad \wedge dy^{\alpha_{\tilde{q}_A+2}}\wedge\cdots\wedge dy^{\alpha_p}. \end{aligned} \tag{3.3}$$

In order to treat both types of the null branes simultaneously, we define the new function as

$$C_A \equiv \partial_v E_A e^{\epsilon_A a_A \varphi - 2\xi - \sum \zeta_\alpha}, \tag{3.4}$$

where

$$\epsilon_A = \begin{cases} +1 \text{ electric field} \\ -1 \text{ magnetic field}. \end{cases} \tag{3.5}$$

3.1 Bosonic part of the solutions and crossing rule

Here we solve the bosonic part basic equations under the ansatzs (3.1), (3.2), and (3.3) as generally as possible and derive the general intersecting rules for N-branes.

By choosing the gauge satisfying

$$(d-1)\eta + \sum_{\alpha=2}^{p} \zeta_\alpha = 0, \tag{3.6}$$

the field equations can be expressed as follows:

$$\begin{aligned}(d-1)(\partial_u\eta - f\partial_v\eta)^2 + \sum_\alpha(\partial_u\zeta_\alpha - f\partial_v\zeta_\alpha)^2 &= -\frac{1}{2}(\partial_u\varphi - f\partial_v\varphi)^2 - \\ &\quad -\sum_A fC_A\partial_v E_A, \end{aligned}\tag{3.7}$$

$$\begin{aligned}2\partial_v(\partial_u\xi - f\partial_v\xi) - \partial_v^2 f + (d-1)(\partial_u\eta - f\partial_v\eta)\partial_v\eta + \\ +\sum_\alpha(\partial_u\zeta_\alpha - f\partial_v\zeta_\alpha)\partial_v\zeta_\alpha &= -\frac{1}{2}(\partial_u\varphi - f\partial_v\varphi)\partial_v\varphi + \\ &\quad +\sum_A \frac{D-q_A-3}{2(D-2)}2C_A\partial_v E_A, \end{aligned}\tag{3.8}$$

$$(d-1)(\partial_v\eta)^2 + \sum_\alpha(\partial_v\zeta_\alpha)^2 = -\frac{1}{2}(\partial_v\varphi)^2, \tag{3.9}$$

$$\partial_v(\partial_u\eta - f\partial_v\eta) = -\sum_A \frac{q_A+1}{2(D-2)}C_A\partial_v E_A, \tag{3.10}$$

$$\partial_v(\partial_u\zeta_\alpha - f\partial_v\zeta_\alpha) = \sum_A \frac{\delta_{\alpha A}}{2(D-2)}C_A\partial_v E_A, \tag{3.11}$$

$$\partial_v(\partial_u\varphi - f\partial_v\varphi) = -\sum_A \frac{1}{2}\epsilon_A a_A C_A\partial_v E_A, \tag{3.12}$$

$$\partial_u C_A = \partial_v C_A = \partial_v(fC_A) = 0, \tag{3.13}$$

where ∂_u and ∂_v are partial derivatives $\partial/\partial u$ and $\partial/\partial v$ in the light-cone coordinates and A denotes the kinds of Nq_A-branes. $\delta_{\alpha A}$ is the constant decided by

$$\delta_{\alpha A} = \begin{cases} D - q_A - 3 \;\; y_\alpha \text{ belonging to } q_A\text{-brane} \\ -(q_A + 1) \quad \text{otherwise.} \end{cases} \tag{3.14}$$

Eqs. (3.7)- (3.11) are the (u,u), (u,v), (v,v), (i,i), (α,α) components of the Einstein equation in eq. (2.2), respectively. From the Maxwell equations (3.13), C_A is shown to be a constant and $\partial_v f = 0$, which suggests $f = f(u)$.

Since C_A is a constant in eq. (3.10), (3.11), and the dilaton equation (3.12),

$$\partial_v \left[(\partial_u - f\partial_v)\eta + \sum_A \frac{q_A + 1}{2(D-2)} C_A E_A \right] = 0\,, \tag{3.15}$$

$$\partial_v \left[(\partial_u - f\partial_v)\zeta_\alpha - \sum_A \frac{\delta_{\alpha A}}{2(D-2)} C_A E_A \right] = 0, \tag{3.16}$$

$$\partial_v \left[(\partial_u - f\partial_v)\varphi + \sum_A \frac{1}{2}\epsilon_A a_A C_A E_A \right] = 0\,, \tag{3.17}$$

and $\partial_v f = 0$ in eq. (3.8) provides

$$\partial_v \left[(\partial_u - f\partial_v)\xi - \sum_A \frac{D - q_A - 3}{2(D-2)} C_A E_A \right] = 0\,. \tag{3.18}$$

In this paper, we concentrate on solutions with the following conditions, which satisfy the above equations automatically:

$$(\partial_u - f\partial_v)\xi = \sum_A \frac{D - q_A - 3}{2(D-2)} C_A E_A\,, \tag{3.19}$$

$$(\partial_u - f\partial_v)\eta = -\sum_A \frac{q_A + 1}{2(D-2)} C_A E_A\,, \tag{3.20}$$

$$(\partial_u - f\partial_v)\zeta_\alpha = \sum_A \frac{\delta_{\alpha A}}{2(D-2)} C_A E_A\,, \tag{3.21}$$

$$(\partial_u - f\partial_v)\varphi = -\sum_A \frac{1}{2}\epsilon_A a_A C_A E_A\,. \tag{3.22}$$

It is worth noting that even though we use eq. (3.19) and eq. (3.22) as concrete examples, in general, integration "constants" which are arbitrary functions in terms of u as in [29] can be considered.

We show these conditions are consistent with the BPS condition, of an extremal solution in supergravity, as we will see in the next subsection. Substituting eqs. (3.20), (3.21), and (3.22) to eq. (3.7), we find

$$\sum_{A,B} \left[M_{AB} \frac{C_A}{2} + \delta_{AB}\partial_v \left(\frac{f}{E_A} \right) \right] \frac{C_B}{2} E_A E_B = 0\,, \tag{3.23}$$

JHEP05(2005)016

where M_{AB} is a constant matrix defined as

$$M_{AB} = (d-1)\frac{(q_A+1)(q_B+1)}{(D-2)^2} + \frac{1}{2}\epsilon_A a_A \epsilon_B a_B \sum_{\alpha=2}^{p} \frac{\delta_{\alpha A}\delta_{\alpha B}}{(D-2)^2}\,. \tag{3.24}$$

Notice that until now our discussion is quite general except for imposing on the ansatzs (3.1), (3.2) and (3.3), and the gauge condition (3.6). From eq. (3.23) an important condition is derived if we require that the functions E_A with different index A are independent, that is, $M_{AB} = 0$ for $A \neq B$. Suppose that Nq_A-brane and Nq_B-brane intersect over $\bar{q}_{AB}+1$ dimensions ($\bar{q}_{AB} < q_A, q_B$). A rule for the crossing dimensions, which is called the crossing rule of the branes is obtained as

$$\bar{q}_{AB} = \frac{(q_A+1)(q_B+1)}{D-2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B\,. \tag{3.25}$$

This crossing rule is the same as that of a previous work considering slightly different situations [29] and as that for the S-brane cases given by [26, 28].

On the other hand, by considering the case $A = B$ in eq. (3.24), we have

$$M_{AA} = \frac{(q_A+1)(D-q_A-3)}{D-2} + \frac{1}{2}a_A^2 \equiv \frac{\Delta_A}{D-2}\,. \tag{3.26}$$

Therefore eq. (3.23) provides

$$E_A = \sqrt{\frac{2(D-2)}{\Delta_A}}\frac{f}{H_A}\,, \tag{3.27}$$

where H_A is a harmonic function on $\{u, v\}$ satisfying

$$\partial_u\partial_v H_A = \partial_v^2 H_A = 0\,, \tag{3.28}$$

which is clear from eq. (3.13).

In the following, we show examples of the solutions satisfying the crossing rule obtained above.

In 11-dimensional supergravity, there is only a 3-form field; thus there is no dilaton φ. Setting $D = 11$ and $a_A = 0$, we find that $\Delta_A = (q_A+1)(8-q_A)$. For the 3-form field, because $n_A = 4$ the electric type field is related to NM2-brane, i.e., $q_A = n_A - 2 = 2$ and $\Delta_A = 18$. Therefore, the solutions with one electrically charged N-brane are then written as

$$\begin{aligned} ds_{11}^2 &= 2e^{2\xi}du(dv+fdu) + e^{2(\zeta_2+\zeta_3)}(dy_2^2+dy_3^2) + e^{2\eta}\sum_{i=1}^{7} dx_i^2\,, \\ F_4 &= -d(f/H_2)\wedge du \wedge dy_2 \wedge dy_3\,, \end{aligned} \tag{3.29}$$

where H_2 is a harmonic function depending on u and v.

On the other hand, the magnetic type field is related to the NM5-brane because $\tilde{q}_A = \tilde{n}_A - 2 = D - n_A - 2 = 5$, which provide $\Delta_A = 18$. Therefore, the solutions with one magnetically charged N-brane are given by

$$\begin{aligned} ds_{11}^2 &= 2e^{2\xi}du(dv + fdu) + \sum_{\alpha=2}^{6} e^{2\zeta_\alpha} dy_\alpha^2 + e^{2\eta} \sum_{i=1}^{4} dx_i^2 \,, \\ *F_4 &= H_5^2 * d(f/H_5) \,, \end{aligned} \tag{3.30}$$

where H_5 is a harmonic function depending on u and v, too.

Of course, it is also possible to introduce the combinations of NM2-branes and NM5-branes. In such cases, the crossing rule obtained in eq. (3.25) plays a very important role. From the crossing rule, all the possible cases for the intersecting dimensions are:

$$M2 \cap M2 \to \bar{q} = 0 \,, \qquad M2 \cap M5 \to \bar{q} = 1 \,, \qquad M5 \cap M5 \to \bar{q} = 3 \,. \tag{3.31}$$

Among them, we obtain $d = 4$, the case with the BPS pp-wave solutions uniquely as follows:

$$ds_{11}^2 = 2e^{2\xi}du(dv + fdu) \sum_{\alpha=2}^{7} e^{2\zeta_\alpha} dy_\alpha^2 + e^{2\eta} \sum_{i=1}^{3} dx_i^2 \,, \tag{3.32}$$

in which the NM2-brane occupies the u, y^2 and y^7 directions and NM5-brane occupies the $u, y^2 \ldots, y^6$ directions. The solution given by eq. (3.32) is especially interesting in that it produces cosmological solutions after the compactification, as we will mention later.

3.2 BPS pp-wave brane solutions

In the previous subsection, we notice only the bosonic part based on the action (2.1), and the solutions obtained are irrelevant to the fermionic part and supersymmetry. Here, since we are interested in supersymmetry and the BPS solutions, we construct concrete BPS solutions. As follows, we can investigate the condition under which supersymmetry remains without writing down the details of the fermionic part. This condition can be attained by considering the supersymmetry transformation of the gravitino ψ_μ, which is called the Killing equation for the Killing spinor ζ [36]. Even though we consider only the 11-dimensional case here for simplicity, it is worth noting that this argument can be extended to the 10-dimensional case easily. In the 11-dimensional case, the vanishing condition of the supersymmetry transformation of the gravitino is given as

$$\delta\psi_\mu = \left[\partial_\mu + \frac{1}{4}\omega^{ab}{}_\mu \gamma_{ab} + \frac{1}{144}(e^a{}_\mu \gamma_a{}^{bcdf} - 8e^a{}_\mu \delta_a^b \gamma^{cdf}) F_{bcdf} \right] \zeta = 0 \,, \tag{3.33}$$

where γ's are the antisymmetrized products of 11-dimensional gamma matrices with unit strength. $e^a{}_\mu$ is a basis of a tetrad, and a spin connection is $\omega^{ab}{}_\mu = e^c{}_\mu \omega^{ab}{}_c$, where $\omega^{ab}{}_\mu$ is defined by

$$\omega_{ab\mu} \equiv \frac{1}{2} e_a{}^\nu (\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}) - \frac{1}{2} e_b{}^\nu (\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}) - \frac{1}{2} e_a{}^\rho e_b{}^\sigma e^c{}_\mu (\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho}) \,. \tag{3.34}$$

JHEP05(2005)016

Now we consider the i-th components of the Killing equations. For the metric given by eq. (3.1), the spin connection, $\omega^{ab}{}_i$, can be written as

$$\omega^{ab}{}_i = -e^{\eta-\xi}\partial_v\eta(\delta^a_u\delta^b_i - \delta^a_i\delta^b_u) + e^{\eta-\xi}\sum_A \frac{q_A+1}{18} f\partial_v \ln H_A(\delta^a_v\delta^b_i - \delta^a_i\delta^b_v)\,, \tag{3.35}$$

where we use the conditions of (3.15). Substituting eq. (3.35) into eq. (3.33), we find the supersymmetric solutions must satisfy

$$\partial_v\eta = \sum_A \frac{q_A+1}{18}\partial_v \ln H_A\,, \tag{3.36}$$

and we can solve this equation as

$$\eta = \sum_A \frac{q_A+1}{18}\ln H_A\,. \tag{3.37}$$

The other conditions of $\delta\psi_\mu = 0$ provide similar conditions; then we finally get

$$\begin{aligned} \xi &= -\sum_A \frac{D-q_A-3}{\Delta_A}\ln H_A\,,\\ \eta &= \sum_A \frac{q_A+1}{\Delta_A}\ln H_A\,,\\ \zeta_\alpha &= -\sum_A \frac{\delta_A}{\Delta_A}\ln H_A\,. \end{aligned} \tag{3.38}$$

The conditions given by eq. (3.38) correspond to the BPS conditions necessary for supersymmetry. The next question is how much supersymmetry remains. Since this depends on what and how many branes are included, we concentrate here on the intersecting N-branes solution given by eq. (3.32), even though the extension to other models is simple.

In this case, by substituting eq. (3.38) into eq. (3.32), the concrete solution is obtained as

$$\begin{aligned} ds^2_{11} = H_2^{1/3}H_5^{2/3}\Big[&(H_2H_5)^{-1}2du(dv+fdu) + (H_2H_5)^{-1}dy_2^2 +\\ &+ H_5^{-1}(dy_3^2+\cdots+dy_6^2) + H_2^{-1}dy_7^2 + dx_1^2 + dx_2^2 + dx_3^2\Big]\,. \end{aligned} \tag{3.39}$$

For this solution, if and only if η satisfies this condition, the Killing equation is satisfied, that is,

$$\begin{aligned} \delta\psi_i = -\frac{1}{6}e^{\eta-\xi}\,[&\partial_v\eta\gamma_{iu}(1+\gamma_{v\alpha_2\alpha_7}) + (\partial_u - f\partial_v)\eta\gamma_{iv}(1+\gamma_{u\alpha_2\alpha_7}) +\\ &+2\partial_v\eta\gamma_{iu}(1-\gamma_{v\alpha_2\cdots\alpha_6}) + 2(\partial_u - f\partial_v)\eta\gamma_{iv}(1-\gamma_{u\alpha_2\cdots\alpha_6})]\,\zeta = 0\,. \end{aligned} \tag{3.40}$$

Thus in order to remain supersymmetrical, it seems that there are four conditions. However, it can be shown that among them, only the following three are independent,

$$(1+\gamma_{v\alpha_2\alpha_7})\zeta = 0\,,\qquad (1+\gamma_{u\alpha_2\alpha_7})\zeta = 0\,,\qquad (1-\gamma_{v\alpha_2\cdots\alpha_6})\zeta = 0\,. \tag{3.41}$$

Since γ's have eigenvalues ± 1 with the same numbers, half of the supersymmetry remains from each condition. Therefore, the Killing spinor considered here has 1/8 supersymmetry, which is related to the four-dimensional $N = 1$ supersymmetry with compactification on a torus.

It is important to discuss the possibility to generate other supersymmetric solutions based on the solutions obtained above. In general, the dimensional reduction of 11-dimensional supergravity provides 10-dimensional theory with two supersymmetries, that is, type IIA supergravity. In this theory, the gravitinos have opposite chiralities (γ eigenvalues), i.e., it is "nonchiral". There is the other 10-dimensional theory with two supersymmetries that cannot be obtained by the reduction or the truncation of the 11-dimensional theory, that is, type IIB supergravity. In this theory, the gravitinos have the same chirality, i.e., it is "chiral". These describe the leading low-energy behaviors of type IIA and type IIB superstring theories respectively. This fact makes the explicit formulations of the supergravity theories of particularly interest.

If we compactify the y^7 coordinate in the solutions above, we obtain the pp-wave solutions in type IIA supergravity theory in which the dilaton φ with coupling constant $\epsilon_A = (3 - q)/2$ appears. In this case, we get

$$\varphi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln H_A \,. \tag{3.42}$$

Since for this case the harmonic rule is satisfied naturally, we can make all the possible solutions in type II supergravity theories by making use of the T and S-dual transformations as in BPS Dp-brane cases.

4. Cosmological solution

In the previous section, we show new classes of the solutions satisfying the BPS conditions in the context of the pp-wave background. However, at this point, even after imposing the BPS conditions, many degrees of freedom remain to fix the solutions concerned with the forms of $H_2(u, v)$, $H_5(u, v)$, $f(u)$. On the other hand, it seems very important to relate the time-dependent solutions that are supposed to describe the early stage of our Universe and the string theory. Therefore, in this section, by choosing the forms of the functions appropriately, we give examples corresponding to the inflation in our Universe, even though we do not mention how they are chosen in detail.

After compactifying the y_α ($\alpha = 2, \cdots, p$) coordinates and conformally transforming into the (d+1)- dimensional Einstein frame, the original Einstein-Hilbert action in the D-dimensional theory can be given by

$$\sqrt{-g_D} R_D \propto \Omega^{d+1} \sqrt{-g_{d+1}} \prod_A H_A^{-q_A \frac{D-q_A-3}{\Delta_A} + (p-q_A-1)\frac{q_A+1}{\Delta_A}} \Omega^{-2} R_{d+1} \,, \tag{4.1}$$

where g_{d+1} and R_{d+1} are the determinant and the scalar curvature with respect to the (d+1)-dimensional metric in the Einstein frame $g^{(d+1)}_{\mu\nu}$. Ω is the conformal factor which

JHEP05(2005)016

relates $g_{\mu\nu}^{(d+1)}$ and the metric directly obtained by the compactification $g_{\mu\nu}^{*(d+1)}$ as $g_{\mu\nu}^{*(d+1)} = \Omega^2 g_{\mu\nu}^{(d+1)}$. We omit the term from the volume of the compactified y_α coordinates.

Since in the (d+1)-dimensional Einstein frame, this term should also scale as $\sqrt{-g_{d+1}} \times R_{d+1}$ Ω can be determined as

$$\Omega = \prod_A H_A^{-\frac{D-q_A-3-(q_A+1)(d-2)}{(d-1)\Delta_A}} . \tag{4.2}$$

Therefore the (d+1)-dimensional metric after compactifying the y_α coordinates can be written as

$$\begin{aligned} ds^2 &= \Xi^{d-2} 2du(dv + f du) + \Xi^{-1} \sum_{i=1}^{d-1} dx_i^2 , \\ \Xi &= \prod_A H_A^{-\frac{2(D-2)}{(d-1)\Delta_A}} . \end{aligned} \tag{4.3}$$

Furthermore, in order to obtain the cosmological solutions, that is, time-dependent ones, we now compactify the $y_1 = (u + v)/\sqrt{2}$ coordinates on S^1, by which the functions H_2, H_5, and f come to depend only on t, and the resulting d-dimensional metric can be written by

$$\begin{aligned} ds_d^2 &= -\Upsilon^{d-3} dt^2 + \Upsilon^{-1} \sum_{i=1}^{d-1} dx_i^2 , \\ \Upsilon &= (1 + f)^{-\frac{1}{d-2}} \prod_A H_A^{-\frac{2(D-2)}{(d-2)\Delta_A}} . \end{aligned} \tag{4.4}$$

In the four-dimensional case, in which we consider the intersecting NM2-brane and NM5-brane especially, the metric can be written as

$$\begin{aligned} ds^2 &= -((1 + f)H_2H_5)^{-1/2} dt^2 + ((1 + f)H_2H_5)^{1/2} \sum_{i=1}^{3} dx_i^2 \\ &= -d\tau^2 + a(\tau)^2 \sum_{i=1}^{3} dx_i^2 , \end{aligned} \tag{4.5}$$

where τ is the cosmic time and $a(\tau) = dt/d\tau$ is the scale factor of our Universe. In this model, the evolution of the scale factor depends on the form of $H_2(t)$, $H_5(t)$, and $f(t)$. One interesting question about the early stage of the Universe is whether the inflationary solutions are consistent with supersymmetry. We find if $((1 + f)H_2H_5) \propto t^4$, the exponentialy expanding universe $a(\tau) \propto e^\tau$ is realized. One example satisying the BPS conditions is H_2, H_5, $f \propto t^{4/3}$. We can also show if $((1 + f)H_2H_5) \propto t^{4(p-1)/p}$, the power law inflationary solutions in which $a(\tau) \propto \tau^{p-1}$ are obtained. It can be easily seen that a solution such as H_2, H_5, $f \propto t^{4(p-1)/3p}$ satisfies the BPS conditions.

In the above we constructed the inflationary solution from the viewpoint of our four-dimensional spacetime by compactifying the y_1 coordinate first. However, we can show the compactification of that coordinate realizes dynamically for the examples considered above. A five-dimensional metric with y_1 direction is given by

$$\begin{aligned} ds^2 &= (H_2H_5)^{-2/3}2du(dv+fdu)+(H_2H_5)^{1/3}\sum_{i=1}^{3}dx_i^2 \\ &= \Omega^2(-d\tau^2+a(\tau)^2\sum_{i=1}^{3}dx_i^2+b(\tau)^2dy^2)\,, \end{aligned} \tag{4.6}$$

where

$$dy = dy_1 - \frac{f}{1+f}dt\,, \qquad b(\tau,y_1)^2 = \sqrt{\frac{(1+f)^3}{H_2H_5}}\,, \tag{4.7}$$

$$\Omega^2 = (1+f)^{-1/2}(H_2H_5)^{-1/6}\,. \tag{4.8}$$

Even though, strictly speaking, the y coordinate does not agree with y_1, it plays the same role in the limit of $f \to 0$, whose scale factor evolves as $a/b \propto (H_2H_5/(1+f))^{1/2}$. Therefore, if we choose the examples in which all functions have the same contributions as mentioned above, for the exponentialy inflation case we find $a/b \propto e^{2\tau/3}$, and for the power-law inflation case we find $a/b \propto \tau^{\frac{2(p-1)}{3}}$, which means that for these cases, the compactification of the y coordinate happens dynamically.

It is worthwhile to relate the above cosmological solution to other solutions based on other types of string theory. When it comes to the $y_2,\ldots,y_7$ coordinates, which are compactified in the above analysis, following a similar idea, it can be shown that the volume element of the y_2 coordinate shrinks more rapidly than that of the $y_3,\ldots,y_7$ coordinates. If we compactify only the y_2 coordinate in 11-dimensional M-theory, we can find the $D2$ and $NS5$-brane's bound state in type IIA string theory. Furthermore, using the T-duality transformation on the y^3 direction, the $D3$ and $NS5$-brane's bound state is obtained. It is shown that the $D3$-brane is rolling in "throat geometry" on the $NS5$ background [37], and the corresponding cosmological solutions are already provided by [38], which is related to the rolling tachyon given by [39], from the point of view of the string theory.

5. Conclusion

In this paper we have examined a system where $D=(d+p)$ dimensional gravity is coupled to a scalar field and arbitrary rank form gauge fields. The corresponding action is so general that it can describe the bosonic part of $D=11$ or $D=10$ supergravities.

The ansatzs employed here are to take the pp-wave background for the metric and the $n=q+2$ gauge field that can describe electric q-brane form and magnetic $(D-p-d-2)$-brane form. In this set up, since they occupy the u coordinate, they are null-branes (N-branes). Since we are interested in time-dependent solutions, we demand all the functions appearing depend only on the null coordinates (u,v).

Under these conditions we constructed intersecting N-brane solutions in a pp-wave background. First we treated the bosonic part by considering the action directly. As a result, we obtained the crossing rule of the intersecting N-branes and the harmonic function rule given by eq. (3.25) and eq. (3.28). We show the crossing rule is the same as that for [29] in the previous work in the context of N-brane solutions. This also agrees with the S-brane cases discussed in [26, 28]. It can also be shown that for the solutions under the above ansatzs, the conditions are given by eq. (3.19), (3.20), (3.21), and (3.22). As concrete examples, we considered 11-dimensional supergravity. In this theory, the electric type 3-form field is related to the NM2-brane, while the magnetic type one is to the NM5-brane. The solutions including a single NM2-brane, a single NM5-brane, and the intersecting NM2-brane and NM5-brane satisfying the crossing rule obtained above can be expressed as eqs. (3.29), (3.30), and (3.32), respectively.

The discussion about the bosonic part is quite general, although there remain many degrees of the freedom about the functions of the metric and the scalar field.

Next, we considered the fermionic part and the conditions under which the solutions satisfy supersymmetry (BPS conditions). We also constructed BPS pp-wave brane solutions in the context of 11-dimensional supergravity. In this analysis, instead of considering the action of the fermionic part directly, we used the supersymmetry transformation of the gravitino, the so-called Killing equation, into which we substituted the results of the bosonic part. If we impose the supersymmetry, the functional forms of the solutions are limited further given by eq. (3.38). From the Killing equation, how much supersymmetry remain in the resultant BPS solutions is determined, which depends on the solutions. For example, in the case of the intersecting N-branes solution given by eq. (3.32), since three independent conditions must be satisfied, 1/8 supersymmetry remains. Even though we have limited the 11-dimensional case only for simplicity, this argument can be extended to the 10-dimensional case easily.

At this point, all the remaining degrees of freedom to fix the solutions are $H_2(u,v)$, $H_5(u,v)$, which are the harmonic functions related with N-branes and $f(u)$, which appeared in the metric ansatz. Even though we have started from 11-dimensional theory, by dimensional reduction and applying S and T-duality trasformations, we could also obtain all standard intersecting BPS brane solutions with pp-wave in 10-dimensional type II theories.

Finally, we applied the BPS solutions to the cosmological setting by compactifying the extra coordinates, even though we have not mentioned the details of the mechanism of the compactification. As is shown by eq. (4.5), the evolution of the scale factor depends on the form of $H_2(t)$, $H_5(t)$, and $f(t)$, which are arbitrary even after imposing the BPS conditions. By choosing the functions appropriately, we obtained the exponentially expanding Universe, as well as the power-law inflationary solutions. Since these inflationary solutions are consistent with supersymmetry, it seems interesting, even though the mechanism to fix the functions is unclear at this time. Furthermore, if we concentrate on the above inflationary solutions, even without compactifying the y_1 direction at first, the compactification of the corresponding coordinate happens dynamically.

It is worth noting that the supersymmetric cosmological metric like this can be hardly obtained. In the case discussed in [29] which examined similar but simpler situations, there is no remaining supersymmetry. Therefore the crucial thing to obtain supersymmetiric solution is not N-brane itself but the generalizetion from [29] to this, that is, all variables should depend on both u and v which relaxes the ansatzs. But it can also be speculated that since in this solution, all time dependence appears in terms of null coordinates, the usual conditions which prohibit the time dependent supersymmetric solutions in terms of time coordinate does not hold. Even though more general discussion for supersymmetric cosmological solution is important, this topic is beyond the scope of this paper.

From the view point of the realistic cosmology, we cannot resist asking whether the standard big-bang Universe is recovered. One interesting scenario is the above inflationary solution becomes unstable as a result of supersymmetry breaking, and the inflation terminates. After supersymmetry breaking, since we need not take account of the BPS conditions, all we have to consider are eqs. (3.19), (3.20), and (3.21), which are by far milder than the BPS conditions. For example, if we choose ζ_α =const provides ξ =const, η =const and f/H_A =const, the cosmic expansion law of the radiation dominated Universe is obtained only in the limit of $H_A = H_A(u) = f(u)$. It is also known that supersymmetry breaking generates potential heat, which describes the reheating process, even though we do not mention the details, and we would like to leave them for future work.

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